On Toeplitz products on Bergman space and
two-weighted inequalities for the Bergman
projection

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In 1978, Aline Bonami and David Békollé characterised the class of weights $w$ on the unit disk $\mathbb{D}$ for which the Bergman projection is bounded on the weighted space $L^2_w(\mathbb{D})$. As in the case of the Hardy space, this weighted theory is closely connected to the theory of Toeplitz operators on the Bergman space, with the boundedness of products of Toeplitz operators corresponding to two-weight boundedness of the Bergman projection for certain weights determined by the symbols of the Toeplitz operators. In the early 90’s, D. Sarason posed conjectures on the characterization of the boundedness of Toeplitz products on Hardy and Bergman spaces. In the Bergman space case, the Sarason condition is just a natural 2-weight version of the David Békollé-Bonami $B_2$ condition.

We solve this conjecture in the negative, showing that the Sarason condition is not sufficient for the boundedness of Toeplitz products on the Bergman space, and give an alternative characterisation. We also obtain sharp estimates for the one-weighted Bergman projection in terms of the $B_2$ and $B_\infty$ characteristics of the weight.

This is joint work with Alexandru Aleman (Lund) and Maria Carmen Reguera (Birmingham).