On Toeplitz products on Bergman space and two-weighted inequalities for the Bergman projection

Sandra Pott (Lund University)

In 1978, Aline Bonami and David Békollé characterised the class of weights w on the unit disk $\mathbb{D}$ for which the Bergman projection is bounded on the weighted space $L_w^2(\mathbb{D})$. As in the case of the Hardy space, this weighted theory is closely connected to the theory of Toeplitz operators on the Bergman space, with the boundedness of products of Toeplitz operaters corresponding to two-weight boundedness of the Bergman projection for certain weights determined by the symbols of the Toeplitz operators. In the early 90's, D. Sarason posed conjectures on the characterization of the boundedness of Toeplitz products on Hardy and Bergman spaces. In the Bergman space case, the Sarason condition is just a natural 2-weight version of the David Békollé-Bonami B_2 condition.

We solve this conjecture in the negative, showing that the Sarason condition is not sufficient for the boundedness of Toeplitz products on the Bergman space, and give an alternative characterisation. We also obtain sharp estimates for the one-weighted Bergman projection in terms of the B_2 and B_∞ characteristics of the weight.

This is joint work with Alexandru Aleman (Lund) and Maria Carmen Reguera (Birmingham).