Functional calculus of Dirac operators and tent spaces.

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In [2], Axelsson, Keith, and McIntosh have shown that results about Riesz transforms, such as the boundedness of the Cauchy integral on Lipschitz curves, and Kato's square root estimates, can be seen as instances of a perturbation result, in L^2, for the holomorphic functional calculus of certain first order differential operators. This perspective has since been proven to be particularly useful for the development of harmonic analysis on manifolds, and in the study of rough boundary value problems. It has been extended from L^2 to L^p in two different ways: using an appropriate extrapolation method in [1], or a set of martingale techniques that provide L^p analogues of the key techniques of [2] in [3].

In this talk, we present an alternative set of L^p techniques based on Hardy spaces rather than martingale methods. This turns out to be simpler and give stronger results, and also points out an interesting phenomenon: the heart of the harmonic analysis in [2] actually extends from L^2 to L^p for all $p \in (1, \infty)$, while the (necessary) restrictions in p only come from an estimate that is trivial in L^2. Our approach is fundamentally based on Coifman-Meyer-Stein's theory of tent spaces, and on the current development of an operator-valued Calderón-Zygmund theory in these spaces.

This is joint work with D. Frey and A. McIntosh.

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[2] A. Axelsson, S. Keith, A. McIntosh, Quadratic estimates and func-

tional calculi of perturbed Dirac operators. *Invent. Math. 163* (2006) 455–497.

[3] T. Hytönen, A. McIntosh, P. Portal, Kato's square root problem in Banach spaces. *J. Funct. Anal. 254* (2008) 675–726.