

Sharp L^p estimates of second order Riesz transforms on discrete abelian groups.

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The real part of the Beurling Ahlfors operator in two dimensions is the difference of the two second order planar Riesz transforms. In 2000, Nazarov and Volberg proved good L^p bounds for this operator. Ten years later, Geiss, Montgomery-Smith, Saksman proved that the Nazarov-Volberg estimate is optimal. Next to the Hilbert transform and the first order Riesz transforms, this has become the second known exact L^p estimate for classical Calderon-Zygmund operators. These estimates enjoy generalisations to Lie groups, but surprising artifacts are observed when the continuity assumption is dropped. The question of optimal L^p estimates for discrete Hilbert transforms (on the integers) is older than Pichorides proof for the continuous Hilbert transform itself. The difficulties that arise are linked to the non-local nature of discrete derivatives that arise when defining these operators. In case of second order Riesz transforms we overcome this difficulty and prove optimal estimates on products of discrete abelian groups, such as the integers or discrete cyclic groups.