A central limit theorem for the Euler characteristic of a Gaussian excursion set

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We study the Euler characteristic of an excursion set of a stationary Gaussian random field.

Let $X : \Omega \times \mathbb{R}^d \to \mathbb{R}$ be a stationary isotropic Gaussian field with trajectories in $C^2(\mathbb{R}^d)$. Let us fix a level $u \in \mathbb{R}$ and consider the excursion set above u, $\{t \in \mathbb{R}^d : X(t) \geq u\}$. We take the restriction to a compact set considering for any bounded rectangle $T \subset \mathbb{R}^d$, $A(T, u) = \{t \in T : X(t) \geq u\}$. The aim of this work is to establish a central limit theorem for the Euler characteristic of $A(T, u)$ as T grows to $\mathbb{R}^d$, as conjectured by R. Adler more than ten years ago. Our result extends to higher dimension what is known in dimension one, since in that case, the Euler characteristic of $A(T, u)$ equals the number of up-crossings of X at level u. Our main tools are the Rice's formula for the Euler characteristic and the expansion into the Wiener-Itô chaos for this functional. This is a joint work with Anne Estrade.