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Wavelet techniques for p-exponent multifractal analysis

The purpose of the multifractal analysis of a function f is to determine the Hausdorff dimensions of the sets of points where a pointwise regularity exponent $h_f(x)$ takes a given value H. The corresponding collection of dimensions $d_f(H)$ is usually referred to as the *multifractal spectrum* of f. In applications, these dimensions are estimated through a Legendre transform of averaged quantities, effectively computable on data. Up to now, multifractal analysis has been mainly developed in the setting where the regularity exponent is the usual Hölder exponent; in this case, this procedure only applies to locally bounded functions, an a priori hypothesis which, unfortunately, is seldom met by real-life signals and images.

We will present an alternative, which is obtained when the usual pointwise Hölder regularity is replaced by the T^p_α regularity, a notion introduced by Calderón and Zygmund in the 1960s, in the context of PDEs. The a priori assumption now is that the data locally belong to L^p, and we will see that it is much more frequently met by experimental data. We will set the mathematical framework supplied by this p-exponent based multifractal analysis, and show applications to stochastic processes, random fields and real-life data, for which the previous approach based on the Hölder exponent did not work. Finally we will discuss selected open problems that are raised by these new developments.

This is joint work with Patrice Abry, Clotilde Melot, Roberto Leonarduzzi, Stéphane Roux, María Eugenia Torres and Herwig Wendt.