On some special isomorphisms of Hardy spaces for certain Schrödinger operators.

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On $\mathbb{R}^d$, $d \geq 3$, we consider the semigroup $\{K_t\}_{t>0}$ of linear operators generated by a Schrödinger operator $L = \Delta - V(x)$, where $V(x)$ is a non-negative locally integrable function.

The Hardy space H^1_L associated with L is defined as

$$H^1_L = \{f \in L^1(\mathbb{R}^d) : \mathcal{M}_L f(x) = \sup_{t>0} |K_t f(x)| < \infty\}$$

with the norm $\|f\|_{H^1_L} = \|\mathcal{M}_L f\|_{L^1}$.

We say that a function w is L-harmonic if it is preserved by the semigroup, that is, $K_t w = w$ for every $t > 0$.

We shall prove that the following two conditions are equivalent:

(1) there is an L-harmonic function w, $0 < \delta \leq w(x) \leq C$, such that the mapping

$$H^1_L \ni f \mapsto wf \in H^1(\mathbb{R}^d)$$

is an isomorphism of the Hardy space H^1_L and the classical Hardy space $H^1(\mathbb{R}^d)$;

(2) the global Kato norm $\|V\|_{\mathcal{K}}$ is finite, where

$$\|V\|_{\mathcal{K}} = \sup_{x \in \mathbb{R}^d} \int_{\mathbb{R}^d} |x-y|^{2-d} V(y)\, dy.$$

Our second result states that in this case the operator $(-\Delta)^{1/2} L^{-1/2}$, which is bounded on $L^1(\mathbb{R}^d)$, turns out to be another isomorphism of the spaces H^1_L and $H^1(\mathbb{R}^d)$.

As corollaries of these results we obtain that the space H^1_L admits:

(3) atomic decomposition with atoms satisfying the support condition $\operatorname{supp} a \subset B$ (for a certain ball B), the size condition $\|a\|_{L^\infty} \leq |B|^{-1}$, and the cancellation condition $\int a(x)w(x)dx = 0$

(4) characterization by the Riesz transforms $R_j = \partial_{x_j} L^{-1/2}$.
The results are joint works with Jacek Zienkiewicz.