Two-sided bounds for L_p-norms of combinations of products of independent random variables

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(based on the joint work with
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I am going to show that for every positive p, the L_p-norm of linear combinations (with scalar or vector coefficients) of products of i.i.d nonnegative random variables with the L_p-norm one is comparable to the l_p norm of the coefficients and the constants are explicite. More precisely, let $X, X_1, X_2, \ldots$ be i.i.d. nonnegative r.v.'s such that $\mathbb{E}X = 1$ and $\mathbb{P}(X = 1) < 1$. Define

$$R_0 := 1 \quad \text{and} \quad R_k := \prod_{j=1}^{k} X_j \ \text{ for } k = 1, 2, \ldots.$$

Then the following theorem holds

Theorem 1. *Let $p > 0$ and $X, X_1, X_2, \ldots$ be an i.i.d. sequence of nonnegative r.v.'s such that $\mathbb{E}X^p < \infty$ and $\mathbb{P}(X = t) < 1$ for all t. Then there are constants $0 < c_{p,X} \le C_{p,X} < \infty$ which depend only on p and the distribution of X such that for any vectors $v_0, v_1, \ldots, v_n$ in a normed space $(F, \| \ \|)$,*

$$c_{p,X} \sum_{i=0}^{n} \|v_i\|^p \mathbb{E}R_i^p \le \mathbb{E}\left\|\sum_{i=0}^{n} v_i R_i\right\|^p \le C_{p,X} \sum_{i=0}^{n} \|v_i\|^p \mathbb{E}R_i^p.$$

As a result the same holds for linear combinations of Riesz products and similar bounds can be proved for partial sums of perpetuities $\sum_{i=1}^{n} R_{i-1}B_i$, where (X_i, B_i) is an i.i.d sequence of random variables with values in $[0, \infty) \times \mathbb{R}$.