HARMONIC ANALYSIS, PROBABILITY & APPLICATIONS

Conference in honor of Aline Bonami

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Eric Amar (Institut Mathématique de Bordeaux)
How a bit of probability eases the study of interpolating sequences in the ball of $\mathbb{C}^n$.

We study interpolating sequences and Carleson measures for the Hardy Sobolev spaces of the unit ball of $\mathbb{C}^n$. We use in particular basic harmonic analysis on finite group and basic probabilities via Rademacher sequences an Khintchine inequalities.
This gives new results on Hardy Sobolev spaces and their multipliers algebra ; some of them completely analogous to the classical case of Hardy spaces of the ball, some of them in complete contrast to them.

Pascal Auscher (Université Paris-Sud)
Bases d'ondelettes dans les structures métriques et quasi-métriques.

La construction des premières bases d'ondelettes, il y a presque 30 ans, a révolutionné les applications au codage et à l'analyse. Le cadre Euclidien a pu être généralisé mais souvent on se contente de systèmes où les décompositions sont non uniques mais encore robustes (les "frames"). Dans un travail avec T. Hytönen, nous avons pu construire des bases orthonormées d'ondelettes dans un cadre aussi général que les espaces de type homogène. Dans l'exposé, on mettra le théorème en perspective des résultats antérieurs et on indiquera les éléments de construction de cette base.

Frédéric Bernicot (CNRS-Université de Nantes)
Sobolev algebra, associated to a semigroup of operator.

In this talk, we are interested in Sobolev spaces defined via an abstract self-adjoint operator generating a semigroup. A natural question is to prove that the intersection of such homogeneous Sobolev space intersected with L^∞ satisfies the algebra property, as in the Euclidean setting. What assumptions, put on the semigroup to prove such inequalities ? We will give some answers, using the paraproducts approach. A chain rule property and some paralinearization formula will be also described. This work is joint with Thierry Coulhon and Dorothee Frey.

Ronald Coifman (Yale University)

Learning Dual contextual/conceptual geometries of Databases/Matrices

We provide an overview of recent developments in geometric Harmonic Analysis methodologies for empirical organization of stochastic data. We focus on data provided as an array or matrix, where we view the rows and columns of the matrix as being in a functional duality, generating joint row/column organizational geometries, and opening the door to automated analytic organizations of Matrices or Databases.

We show that the various metrics generated can be viewed as generalizing Earth mover metrics, in which geometry and statistics are mutually supportive. In particular, we introduce methodologies extending Harmonic analysis and "signal processing" on data matrices, enabling functional regression, prediction, denoising, compression, fast numerics, and so on. We illustrate these ideas to organize and map out in an automatic and purely data driven fashion, text documents, psychological questionnaires, medical profiles, physical sensor data, financial data.

As an application to Mathematics , we show that this organization enables the constructions of a dual phase space geometry of eigenfunctions of Laplace Beltrami operators (where the eigenvectors are the rows (questions) and the points are the columns), in this dual geometry we have a corresponding Heisenberg principle.

Ewa Damek (Uniwersytet Wrocławski, Poland)
Two-sided bounds for L_p-norms of combinations of products of independent random variables

I am going to show that for every positive p, the L_p-norm of linear combinations (with scalar or vector coefficients) of products of i.i.d nonnegative random variables with the L_p-norm one is comparable to the l_p norm of the coefficients and the constants are explicite. More precisely, let $X, X_1, X_2, \ldots$ be i.i.d. nonnegative r.v.'s such that $\mathbb{E}X = 1$ and $\Pr(X = 1) < 1$. Define

$$R_0 := 1 \quad \text{and} \quad R_k := \prod_{j=1}^{k} X_j \ \text{ for } k = 1, 2, \ldots.$$

Then the following theorem holds

Theorem 1 *Let $p > 0$ and $X, X_1, X_2, \ldots$ be an i.i.d. sequence of nonnegative r.v.'s such that $\mathbb{E}X^p < \infty$ and $\Pr(X = t) < 1$ for all t. Then there are constants $0 < c_{p,X} \le C_{p,X} < \infty$ which depend only on p and the distribution of X such that for any vectors $v_0, v_1, \ldots, v_n$ in a normed space $(F, \| \ \|)$,*

$$c_{p,X} \sum_{i=0}^{n} \|v_i\|^p \mathbb{E}R_i^p \le \mathbb{E}\left\| \sum_{i=0}^{n} v_i R_i \right\|^p \le C_{p,X} \sum_{i=0}^{n} \|v_i\|^p \mathbb{E}R_i^p.$$

As a result the same holds for linear combinations of Riesz products and similar bounds can be proved for partial sums of perpetuities $\sum_{i=1}^{n} R_{i-1}B_i$, where (X_i, B_i) is an i.i.d sequence of random variables with values in $[0, \infty) \times \mathbb{R}$. Based on joint work with Rafał Latała, Piotr Nayar - University of Wasaw and Tomasz Tkocz - University of Warwick.

Jacek Dziubański (Uniwersytet Wrocławski, Poland)
On some special isomorphisms of Hardy spaces for certain Schrödinger operators

On $\mathbb{R}^d$, $d \geq 3$, we consider the semigroup $\{K_t\}_{t>0}$ of linear operators generated by a Schrödinger operator $L = \Delta - V(x)$, where $V(x)$ is a non-negative locally integrable function.

The Hardy space H^1_L associated with L is defined as

$$H^1_L = \{f \in L^1(\mathbb{R}^d) : \mathcal{M}_L f(x) = \sup_{t>0} |K_t f(x)| < \infty\}$$

with the norm $\|f\|_{H^1_L} = \|\mathcal{M}_L f\|_{L^1}$.

We say that a function w is L-harmonic if it is preserved by the semigroup, that is, $K_t w = w$ for every $t > 0$.

We shall prove that the following two conditions are equivalent:

(1) there is an L-harmonic function w, $0 < \delta \leq w(x) \leq C$, such that the mapping

$$H^1_L \ni f \mapsto wf \in H^1(\mathbb{R}^d)$$

is an isomorphism of the Hardy space H^1_L and the classical Hardy space $H^1(\mathbb{R}^d)$;

(2) the global Kato norm $\|V\|_{\mathcal{K}}$ is finite, where

$$\|V\|_{\mathcal{K}} = \sup_{x\in\mathbb{R}^d} \int_{\mathbb{R}^d} |x-y|^{2-d} V(y)\, dy.$$

Our second result states that in this case the operator $(-\Delta)^{1/2} L^{-1/2}$, which is bounded on $L^1(\mathbb{R}^d)$, turns out to be another isomorphism of the spaces H^1_L and $H^1(\mathbb{R}^d)$.

As corollaries of these results we obtain that the space H^1_L admits:

(3) atomic decomposition with atoms satisfying the support condition $\operatorname{supp} a \subset B$ (for a certain ball B), the size condition $\|a\|_{L^\infty} \leq |B|^{-1}$, and the cancellation condition $\int a(x)w(x)dx = 0$

(4) characterization by the Riesz transforms $R_j = \partial_{x_j} L^{-1/2}$.

The results are joint works with Jacek Zienkiewicz.

PIOTR GRACZYCK (Université d'Angers)

Non-colliding particle systems.

This is a joint work with J. Malecki (Wroclaw). We study the existence and unicity questions for SDEs describing non-colliding particle systems and properties of these particle systems. A. Bonami studied such systems with F. Bouchut, E. Cépa and D. Lépingle (J. Funct. Anal. 165(1999)).

STÉPHANE JAFFARD (Université Paris-Est Créteil))

Wavelet techniques for p-exponent multifractal analysis

The purpose of the multifractal analysis of a function f is to determine the Hausdorff dimensions of the sets of points where a pointwise regularity exponent $h_f(x)$ takes a given value H. The corresponding collection of dimensions $d_f(H)$ is usually referred to as the *multifractal spectrum* of f. In applications, these dimensions are estimated through a Legendre transform of averaged quantities, effectively computable on data. Up to now, multifractal analysis has been mainly developed in the setting where the regularity exponent is the usual Hölder exponent; in this case, this procedure only applies to locally bounded functions, an a priori hypothesis which, unfortunately, is seldom met by real-life signals and images.

We will present an alternative, which is obtained when the usual pointwise Hölder regularity is replaced by the T^p_α regularity, a notion introduced by Calderón and Zygmund in the 1960s, in the context of PDEs. The a priori assumption now is that the data locally belong to L^p, and we will see that it is much more frequently met by experimental data. We will set the mathematical framework supplied by this p-exponent based multifractal analysis, and show applications to stochastic processes, random fields and real-life data, for which the previous approach based on the Hölder exponent did not work. Finally we will discuss selected open problems that are raised by these new developments.

This is joint work with Patrice Abry, Clotilde Melot, Roberto Leonarduzzi, Stéphane Roux, María Eugenia Torres and Herwig Wendt.

Abderrazek Karoui (University of Carthage, Tunisia)
Spectral decay of the sinc kernel operator and approximation by Prolate Spheroidal Wave Functions.

The purpose of this talk is to give a summary of some of the recent results obtained in collaboration with A. Bonami. These results deal with the accurate explicit estimates of the PSWFs and their associated eigenvalues. Note that for a fixed real number $c>0$, called the bandwidth, the prolate spheroidal wave functions (PSWFs) $(\psi_{n,c})_{n\geq 0}$ have been largely studied and used in various classical as well as in some emerging applications in different scientific areas such as Signal Processing and Physics. The PSWFs were first known as the bounded eigenfunctions of the Sturm-Liouville operator's L_c defined on $C^2([-1,1])$ by $L_c(\psi) = -(1-x^2)\frac{d^2\psi}{d\,x^2} + 2x\frac{d\psi}{d\,x} + c^2x^2\psi$. A breakthrough in the subject of the PSWFs is due to D. Slepian, H. Landau and H. Pollack, who have shown that the $\psi_{n,c}$ are also the eigenfunctions of the integral operators F_c and $Q_c = \frac{c}{2\pi}F_c^*F_c$, defined on $L^2([-1,1])$ by

$$F_c(f)(x) = \int_{-1}^{1} e^{i\,c\,x\,y} f(y)\,dy, \quad Q_c(\psi)(x) = \int_{-1}^{1} \frac{\sin c(x-y)}{\pi(x-y)}\,\psi(y)\,dy. \tag{1}$$

The PSWFs exhibit the desirable properties to form an orthogonal basis of $L^2([-1,1])$, and an orthonormal basis of the Paley-Wiener space $B_c = \{f \in L^2(\mathbf{R}),\ \text{Support}\,\widehat{f} \subset [-c,c]\}$. We let $(\lambda_n(c))_n$ denotes the infinite sequence of the eigenvalues of Q_c, arranged in the decreasing order. Many of the applications of the PSWFs heavily rely on their explicit analytic properties as well as on the precise behaviour and decay rate of the corresponding eigenvalues $(\lambda_n(c))_{n\geq 0}$. Although, there exists a rich literature on the numerical computation and asymptotic behaviour of the $\psi_{n,c}$ and $\lambda_n(c)$, very little is known about explicit estimates.

In this talk, we first give new bounds of the $\psi_{n,c}$ and the eigenvalues $\chi_n(c)$ of the differential operator L_c. Then, under a condition given in term of the quantity $q = c^2/\chi_n(c)$, we prove that $\psi_{n,c}$ is uniformly approximated on $[-1,1]$ by a single function involving the function J_0 : the Bessel function of first kind and order zero. As a consequence of this uniform approximation, we provide a sharp exponential decay rate of the $\lambda_n(c)$ which is given by

$$\widetilde{\lambda}_n(c) = \frac{1}{2}\exp\left(-\frac{\pi^2}{2}n\int_{\Phi\left(\frac{2c}{\pi n}\right)}^{1} \frac{1}{t(\mathbf{E}(t))^2}\,dt\right). \tag{2}$$

Here, $\mathbf{E}(x) = \int_0^1 \sqrt{\frac{1-x^2t^2}{1-t^2}}\, dt$ is the elliptic Legendre integral of the second kind and $\Phi = \Psi^{-1}$, where $\Psi(x) = \frac{x}{\mathbf{E}(x)}$, $0 \leq x \leq 1$. As an application, we give the quality of approximation by the PSWFs in the spaces of band-limited, almost band-limited functions as well as on the Sobolev spaces $H^s([-1,1]), s > 0$. The different results of this talk, are illustrated by some numerical examples.

José León (Universidad Central de Venezuela)
A central limit theorem for the Euler characteristic of a Gaussian excursion set

We study the Euler characteristic of an excursion set of a stationary Gaussian random field.

Let $X : \Omega \times \mathbb{R}^d \to \mathbb{R}$ be a stationary isotropic Gaussian field with trajectories in $C^2(\mathbb{R}^d)$. Let us fix a level $u \in \mathbb{R}$ and consider the excursion set above u, $\{t \in \mathbb{R}^d : X(t) \geq u\}$. We take the restriction to a compact set considering for any bounded rectangle $T \subset \mathbb{R}^d$, $A(T,u) = \{t \in T : X(t) \geq u\}$. The aim of this work is to establish a central limit theorem for the Euler characteristic of $A(T,u)$ as T grows to $\mathbb{R}^d$, as conjectured by R. Adler more than ten years ago. Our result extends to higher dimension what is known in dimension one, since in that case, the Euler characteristic of $A(T,u)$ equals the number of up-crossings of X at level u. Our main tools are the Rice's formula for the Euler characteristic and the expansion into the Wiener-Itô chaos for this functional. This is a joint work with Anne Estrade.

Shobha Madan, Indian Institute of Technology, Kanpur
On the structure of Tiles and Spectra

In this talk we will restrict ourselves to the 1-dimensional case of the (so far unsolved) Conjecture of Fuglede, relating Tiles and Spectra. Lagarias and Wang (1996) proved that Tiling sets are periodic and rational. The periodicity of the spectrum was proved more recently. Finally we give a brief commentary on the rationality of the spectrum, which seems to be a harder problem.

Yves Meyer (Académie des sciences et ENS Cachan)
Almost periodic patterns and quasicrystals.

Quasicrystals are almost periodic structures. This belongs to the folklore but depends on the definition of almost periodicity which is being used. This issue has already been discussed in a paper published in the African Diaspora Journal of Mathematics (Volume 13, Number 1, pp. 1–45 (2012)). Here we give another approach based on the analysis of patches of a quasicrystal. Patches became one of the most powerfull tools in image processing. This is a joint work with Pierre-Antoine Guihéneuf.

Cyrille Nana (Université de Buea)
L^p-estimates of the Bergman projection in some homogeneous domains of $\mathbb{C}^n$.

Ivan Nourdin (Université du Luxembourg)
Méthode de Stein et inégalités de Sobolev logarithmique et de transport.

Aline est mondialement célèbre pour avoir été la première à découvrir l'inégalité d'hypercontractivité (dans le cas booléen) en 1968, résultat qui est connu aujourd'hui sous le nom de théorème de Bonami-Beckner. Quelques années plus tard Leonard Gross publia un papier fondamental, dans lequel l'équivalence entre l'inégalité de Sobolev logarithmique et celle d'Aline était présentée. De nombreuses extensions et ramifications ont ensuite été mises en évidence, notamment celle obtenues par Roger-Dominique Bakry et Michel Emery dans les années 80, qui fut d'ailleurs un des ingrédients importants de la preuve de conjecture de Poincaré par Grigori Perelman en 2002. Mon exposé aura pour but de présenter les résultats principaux d'un article récent (rédigé en collaboration avec Michel Ledoux et Giovanni Peccati) autour de ce sujet. Dans notre travail, nous avons découvert une nouvelle classe d'inégalités fonctionnelles, qui relient l'entropie relative H, l'écart de Stein S (un nouvel ingrédient que nous avons introduit), l'information de Fisher I et/ou la distance quadratique de Wasserstein W. Dans le cas gaussien, nous améliorons systématiquement l'inégalité de Sobolev logarithmique, ainsi qu'une inégalité de transport due à Michel Talagrand. Et dans certaines situations, il arrive que notre inégalité soit aussi meilleure que la célèbre inégalité HWI de Felix Otto et Cédric Villani.

Pierre Portal (Australian National Univ., Australie)
Functional calculus of Dirac operators and tent spaces.

In [2], Axelsson, Keith, and McIntosh have shown that results about Riesz transforms, such as the boundedness of the Cauchy integral on Lipschitz curves, and Kato's square root estimates, can be seen as instances of a perturbation result, in L^2, for the holomorphic functional calculus of certain first order differential operators. This perspective has since been proven to be particularly useful for the development of harmonic analysis on manifolds, and in the study of rough boundary value problems. It has been extended from L^2 to L^p in two different ways: using an appropriate extrapolation method in [1], or a set of martingale techniques that provide L^p analogues of the key techniques of [2] in [3].

In this talk, we present an alternative set of L^p techniques based on Hardy spaces rather than martingale methods. This turns out to be simpler and give stronger results, and also points out an interesting phenomenon: the heart of the harmonic analysis in [2] actually extends from L^2 to L^p for all $p \in (1, \infty)$, while the (necessary) restrictions in p only come from an estimate that is trivial in L^2. Our approach is fundamentally based on Coifman-Meyer-Stein's theory of tent spaces, and on the current development of an operator-valued Calderón-Zygmund theory in these spaces.

This is joint work with D. Frey and A. McIntosh.

[1] P. Auscher, On necessary and sufficient conditions for L^p-estimates of Riesz transforms associated to elliptic operators on $\mathbb{R}^n$ and related estimates. *Mem. Amer. Math. Soc. 871* (2007).

[2] A. Axelsson, S. Keith, A. McIntosh, Quadratic estimates and functional calculi of perturbed Dirac operators. *Invent. Math. 163* (2006) 455–497.

[3] T. Hytönen, A. McIntosh, P. Portal, Kato's square root problem in Banach spaces. *J. Funct. Anal. 254* (2008) 675–726.

Sandra Pott (Lund University)

On Toeplitz products on Bergman space and two-weighted inequalities for the Bergman projection

In 1978, Aline Bonami and David Békollé characterised the class of weights w on the unit disk $\mathbb{D}$ for which the Bergman projection is bounded on the weighted space $L^2_w(\mathbb{D})$. As in the case of the Hardy space, this weighted theory is closely connected to the theory of Toeplitz operators on the Bergman space, with the boundedness of products of Toeplitz operaters corresponding to two-weight boundedness of the Bergman projection for certain weights determined by the symbols of the Toeplitz operators. In the early 90's, D. Sarason posed conjectures on the characterization of the boundedness of Toeplitz products on Hardy and Bergman spaces. In the Bergman space case, the Sarason condition is just a natural 2-weight version of the David Békollé-Bonami B_2 condition.

We solve this conjecture in the negative, showing that the Sarason condition is not sufficient for the boundedness of Toeplitz products on the Bergman space, and give an alternative characterisation. We also obtain sharp estimates for the one-weighted Bergman projection in terms of the B_2 and B_∞ characteristics of the weight.

This is joint work with Alexandru Aleman (Lund) and Maria Carmen Reguera (Birmingham).

E. Tchoundja (Université de Yaoundé)

Weak Factorization and Hankel Operators on Hardy-Orlicz and Bergman-Orlicz spaces.

The holomorphic Hardy-Orlicz spaces are natural generalizations of the usual Hardy spaces in the unit ball of $\mathbb{C}^n$. Weak factorization theorem for function in Hardy spaces has been obtained by R. Coifman, R. Rochberg and G. Weiss. We give several generalization of this weak factorization theorem for functions in $\mathcal{H}^\Phi(\mathbb{B}^n)$, with concave growth function, in terms of products of Hardy-Orlicz functions with convex or concave growth functions. We then apply the result to characterize the symbols of (small)Hankel operators that extend into bounded operators from the Hardy-Orlicz $\mathcal{H}^{\Phi_1}(\mathbb{B}^n)$ into $\mathcal{H}^{\Phi_2}(\mathbb{B}^n)$ in the unit ball of $\mathbb{C}^n$, in the case where the growth functions Φ_1 and Φ_2 are either concave or convex. The analogue problem in the case of Bergman-Orlicz spaces is studied, we will give partial results in this situation. This talk is based on joint work with B. Sehba.

Talk cancelled

STEFANIE PETERMICHL (Université de Toulouse)
Sharp L^p estimates of second order Riesz transforms on discrete abelian groups.

The real part of the Beurling Ahlfors operator in two dimensions is the difference of the two second order planar Riesz transforms. In 2000, Nazarov and Volberg proved good L^p bounds for this operator. Ten years later, Geiss, Montgomery-Smith, Saksman proved that the Nazarov-Volberg estimate is optimal. Next to the Hilbert transform and the first order Riesz transforms, this has become the second known exact L^p estimate for classical Calderon-Zygmund operators. These estimates enjoy generalisations to Lie groups, but surprising artifacts are observed when the continuity assumption is dropped. The question of optimal L^p estimates for discrete Hilbert transforms (on the integers) is older than Pichorides proof for the continuous Hilbert transform itself. The difficulties that arise are linked to the non-local nature of discrete derivatives that arise when defining these operators. In case of second order Riesz transforms we overcome this difficulty and prove optimal estimates on products of discrete abelian groups, such as the integers or discrete cyclic groups.

BÉATRICE VEDEL (Université de Bretagne-Sud)
Hyperbolic wavelet analysis of anisotropic textures : global regularity and multifractal formalism.

Joint work with P. Abry, M. Clausel, S. Jaffard and S. Roux.

Anisotropic images -that is images having different geometric characteristics along different directions - naturally appear in various areas (biomedical, hydrology, geostatistics and spatial statistics...) and in many applications, the detection and characterization of the anisotropy is an important issue. We prove that, up to the lost of a logarithmic correction on the wavelet size estimates, the hyperbolic wavelet basis -which is not taylored to one particular anisotropy - gives characterizations of anisotropic Besov spaces $B^{s,\alpha}_{p,q}(\mathbb{R}^2)$. This result leads to an efficient algorithm to detect global anisotropic characteristics, that we have tested, among others, on anisotropic gaussian fields (OSGRF). A similar result is shown for anisotropic pointwise regularity. Further, we relate local regularity features to global quantities on the hyperbolic wavelet coefficients of the analyzed texture. We then pave the way to a new multifractal attributes for images, allowing to describe simultaneously possible complex scales invariances properties and anisotropic characteristics.

Brett Wick (Georgia Institute of Technology (GaTech))
Carleson Measures for Spaces of Analytic Functions.

Carleson measures are a fundamental object in the study of function spaces in one and several variables. While these measures can be defined purely in terms of function theory, in applications of these measures one wants a more 'usable' equivalent characterization of these measures. Finding this equivalent characterization will connect us to the study of weighted inequalities in harmonic analysis and bring in a large family of tools by which to study these objects and questions. In this talk we will discuss recent results where the characterization of these measures is obtained by testing conditions. In particular we will discuss how this plays a role in understanding the Carleson measure for Besov-Sobolev spaces and for model spaces.

Jim Wright (University of Edinburgh)
Affine invariant harmonic analysis

We survey recent progress on this developing area of harmonic analysis.